

Entropy Enhancement and Black Hole Microstates

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Abstract

We study fluctuating two-charge supertubes in three-charge geometries. We show that the entropy of these supertubes is determined by their locally-defined *effective* charges, which differ from their asymptotic charges by terms proportional to the background magnetic fields. When supertubes are placed in deep, scaling microstate solutions, these effective charges can become very large, leading to a much larger entropy than one naively would expect. Since fluctuating supertubes source smooth geometries in certain duality frames, we propose that such an entropy enhancement mechanism might lead to a black-hole like entropy coming entirely from configurations that are smooth and horizonless in the regime of parameters where the classical black hole exists.

1 Introduction

There is a significant body of evidence that supports the idea that, within string theory, one can resolve BPS black hole singularities in terms of regular, horizonless *microstate geometries*. These geometries describe the microstates of black holes in the same regime of parameters where the classical black hole exists (see [1, 2, 3] for reviews). One of the primary issues in proving this idea is whether the known microstate geometries represent typical black hole microstates or whether they are somehow confined to a peculiar atypical sub-sector of the Hilbert space.

To refine this issue, one should first note that in the large- N limit, bulk classical geometries describe, to arbitrary accuracy, bulk quantum states that are dual to coherent states within the Hilbert space of states of the dual CFT. Coherent states can always be used to provide a basis for the Hilbert space, but this may not be so for the “semi-classical states” described by classical geometries. Indeed, as one finds with two-charge geometries, some of the boundary coherent states in such a basis will be dual to geometries that have string-scale features and for which the supergravity approximation breaks down or is, at best, a heuristic guide.

These issues are, however, not directly relevant if one’s goal is to argue that the entropy of a black hole comes from horizon-sized, horizonless configurations that have unitary scattering and hence no information loss: For this, the relevant question is whether the states corresponding to such smooth microstate geometries are suitably dense within the Hilbert space of states. Indeed, in the vicinity a single, smooth microstate geometry that is well-described in supergravity there might exist a vast (but controlled) number of quantum microstates that have the same essential features (such as size, absence of horizon and sub-leading dipole fields). Thus the classical microstate geometry would act as a representative of these quantum microstates.

Hence, in counting semi-classical microstate geometries the first goal is to get the correct dependence of the number geometries as a function of the charges. For BPS black holes in five dimensions, this means one must have:

$$S \sim \log(N) \sim \sqrt{Q_1 Q_2 Q_3}. \quad (1)$$

If $N_{quantum}$ and N_{geom} respectively represent the number of quantum states and the number of semi-classical, microstate geometries that are valid in the supergravity approximation, then one can recover (1), if $\log(N_{geom})$ and $\log(N_{quantum})$ have the same growth to leading order in the charges¹.

A subsequent goal is to get the correct coefficient, $S = 2\pi\sqrt{Q_1 Q_2 Q_3}$, which amounts to predicting the correct central charge for the underlying conformal field theory. On the other hand, if one restricts oneself to a finite fraction of the degrees of freedom (with, perhaps, a lower central charge) and obtains horizon-sized, horizonless black-hole microstates with unitary scattering, it seems very implausible that restoring the rest of the degrees of freedom will drastically change the macroscopic features of these microstates. In particular, it is very unlikely that restoring such degrees of freedom will generate a horizon.

Thus, establishing that black holes in string theory are ensembles of horizonless configurations with unitary scattering is not as demanding as it might, at first, seem, and could reduce to showing that a semi-classical counting of smooth, horizonless, classical microstate geometries

¹In this sense, “suitably dense” can, in fact, amount to an extremely sparse relative population.

gives a black-hole-like, or macroscopic, entropy (1). Indeed, it is our purpose here to display a mechanism by which smooth microstates of such a large entropy can arise.

In [4, 5] it was argued that the deep, or scaling, microstate geometries are the gravitational duals of states that belong to the “typical sector” of the D1-D5-P CFT. This was based upon the fact that a typical excitation of the gravitational system had precisely the correct energy to be the dual of an excitation in the sector of the CFT that contributes maximally to the entropy. In particular, the gravitational red-shift of the throat provides a critical factor in arriving at the proper excitation energies. Thus deep, or scaling geometries [4, 5, 6] will be one of the crucial ingredients in accounting for the entropy of black holes using microstate geometries.

Another important ingredient in our discussion will be the fact that two-charge supertubes [7], which can have arbitrary shapes, give smooth supergravity solutions in the duality frame in which they have D1 and D5 charges [8, 9]. This has been very useful in matching the entropy of two-charge smooth supergravity solutions to that of the dual CFT and served as one of the motivations of the formulation of the fuzzball proposal. However, even if supertubes can have arbitrary shapes, and hence a lot of entropy, their naive quantization cannot hope to account for the entropy of a black hole with a non-trivial, macroscopic horizon (1). Indeed, as found in [10, 11, 12], since supertubes only carry two charges, their entropy scales like:

$$S \sim \sqrt{Q_1 Q_2}. \quad (2)$$

The new insight here comes from considering supertubes in the background of a scaling geometry. We generalize the analysis of [10], and use the supertube DBI–WZ action to count states of quantized supertubes in non-trivial background geometries. We find that, for the purposes of entropy counting, the supertube charges Q_I that appear in (2) are replaced by the local effective charges of the supertube, Q_I^{eff} , which are a combination of the supertube charges and terms coming from the interaction between the supertube magnetic dipole moment and the background magnetic dipole fields.

If there are strong magnetic fluxes in the background, as there are in a deep, bubbled microstate geometries, these effective charges can be *much larger* than the asymptotic charges of the configuration, and can thus lead to a very large entropy enhancement! Indeed, one finds that if the supertube is put in certain deep scaling solutions, the effective charges can diverge if the supertube is suitably localized or if the length of the throat goes to infinity. Of course, this divergence is merely the result of not considering the back-reaction of the wiggly supertube on its background: Once this back-reaction is taken into account, the supertube will delocalize and the fine balance needed to create extremely deep scaling solutions might be destroyed if the tube wiggles too much.

Hence, we expect a huge range of possibilities in the the semi-classical configuration space, from very shallow solutions to very deep solutions. In very shallow solutions, the supertubes can oscillate a lot, but they will not have their entropy enhanced and for very deep solutions the supertube will have vastly enhanced charges but, if the solution is to remain deep, the supertube will be very limited in its oscillations. One can thus imagine that the solutions with most of the entropy will be intermediate, neither too shallow (so as to obtain effective charge enhancement), nor too deep (to allow the supertube to fluctuate significantly). To fully support this intuition one will need to construct the full back-reacted supergravity solution for wiggly supertubes in

bubbling three-charge backgrounds. Even though we do not yet have such solutions, it is possible to use the *AdS*-CFT correspondence to estimate the depth of the bulk microstate solutions dual to states in the typical sector of the dual CFT [4]. We will use this result to determine the depth of the typical throat and then argue that the effective supertube charges corresponding to this throat depth yields an enhanced supertube entropy that is macroscopic (1).

It is interesting to note that entropy enhancement is not just a red-shift effect: There is no entropy enhancement unless there are strong background magnetic fluxes. A three-charge BPS black hole will not enhance the entropy of supertubes: it is only solutions that have dipole charges, like bubbled black holes or black rings that can generate supertube entropy enhancement.

The last ingredient that we use is the generalized spectral flow transformation [13]² that enables us to start from a simple, bubbled black hole microstate geometry [16, 17] and generate a bubbled geometry in which one or several of the Gibbons-Hawking (GH) centers are transformed into smooth two-charge supertubes. Indeed, from a six-dimensional perspective (in a IIB duality frame in which the solution has D1-D5-P charges) this mapping is simply a coordinate transformation. One can then study the particular class of fluctuating microstate geometries that result from allowing the supertube component to oscillate in the deep bubbled geometries. The naive expectation is that one would recover an entropy of the form (2) but, as we indicated, the Q_I are replaced by the enhanced Q_I^{eff} , and the entropy of these supertubes can become “macroscopic” in that it corresponds to the entropy of a black hole with a macroscopic horizon. One can then undo the spectral flow to argue that this entropy is present in the BPS fluctuations of three-charge bubbling solutions in *any* duality frame. In fact, spectrally flowing configurations with oscillating supertubes into other duality frames is not strictly speaking necessary for the purpose of illustrating entropy enhancement and arguing that smooth solutions can give macroscopically large entropy. After all, one could do the full analysis in the D1-D5-P duality frame and consider smooth black hole microstates containing both GH centers and supertubes. Nevertheless, since such solutions have not been studied in the past in great detail, it is easiest to construct them by spectrally flowing multi-center GH solutions, which have been studied much more and are better understood.

The fluctuations we consider do not represent the most general, regular fluctuation of the geometry, but as we outlined earlier, this is not the point: They represent a sub-sector of the possible fluctuations whose Hilbert space has entropy that grows much faster than (2) and indeed might grow as fast as (1). Thus we believe that these microstate geometries may be capturing generic states of the CFT for black holes and black rings with non-zero horizon area and capturing enough of them to account for that horizon area, up to overall numerical factors. The fact that we are only looking at a special class of fluctuations means that we are necessarily restricting the degrees of freedom of the fluctuations and so one would not expect, at the first pass, to recover the correct numerical factors in (1). The important progress here is that we see how microstate geometries may indeed capture enough entropy to account for macroscopic horizons and for their dependence upon charges.

It is also interesting to note that a similar conclusion – that deep, scaling, horizonless configurations can give a macroscopic (black-hole-like) entropy – was also reached in [18] and [19]. In [18] this was done by considering D0 branes in a background of D6 branes with world-volume

²See [14, 15] for relevant earlier work.

fluxes, in the regime of parameters where the D0 branes do not back-react. In [19], a similar result was obtained by studying the quiver quantum mechanics of multiple D6 branes, in the regime where the branes do not back-react, but form a finite-sized configuration. Since these computations were performed in a regime in which the gravitational back-reaction of all or some of the branes is neglected, it is not clear how the configurations that give the black hole entropy will develop in the regime of parameters in which the classical black hole exists, and *all* the branes back-react on the geometry. Their size will continue increasing at the same rate as the would-be black hole horizon, and since they are made from primitive branes, it is very unlikely they will develop a horizon. Hence these two calculations do suggest that the black hole entropy comes from horizonless configurations. However, since the D0 branes give rise to naked singularities, the naive strong-coupling extrapolation of these microstate configurations will not be reliable when the classical black hole exists.

The microstates that we consider here are also counted in a regime of parameters in which some of their components, *i.e.* the supertubes, are treated as probes and described by their DBI–WZ action, and hence do not back-react on the geometry. However, unlike the configurations mentioned above, we understand very well what the supertubes become in the regime of parameters where the black hole exists: They give rise to smooth horizonless microstate solutions. Indeed, as we will show in [20], the DBI–WZ description of supertubes gives configurations that in the D1–D5–P duality frame are smooth in supergravity. Hence, our entropy calculation is expected to extend to the regime of parameters where the classical black hole exists.

In section 2 we review the form of general BPS supergravity solutions with a Gibbons–Hawking (GH) base and in section 3 we compute the entropy of two-charge supertubes in such solutions. In Section 4 we discuss the entropy enhancement mechanism, and in section 5 we consider how the effective charges that give the enhanced entropy behave in deep scaling solutions. Section 6 contains conclusions.

2 Fluctuating supertubes in non-trivial backgrounds

Three-charge bubbling solutions that have the same charges and dipole moments as black holes and black rings are determined by specifying a four-dimensional base space, and solving a set of linear equations to determine the warp factors, and the other parameters of the solution [21].

In the duality frame where the charges of the solutions correspond to D0 branes, D4 branes and F1 strings, the metric and the dilaton have the form:

$$\begin{aligned} ds_{10}^2 &= -\frac{1}{Z_3\sqrt{Z_1Z_2}}(dt+k)^2 + \sqrt{Z_1Z_2}ds_4^2 + \frac{\sqrt{Z_1Z_2}}{Z_3}dx_5^2 + \sqrt{\frac{Z_1}{Z_2}}(dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2), \\ \Phi &= \frac{1}{4}\log\left(\frac{Z_1^3}{Z_2Z_3^2}\right), \end{aligned} \tag{3}$$

where we parameterize the S^1 of the F1 string by x^5 , and the T_4 of the D4 world-volume by x_i , $i = 1, \dots, 4$. The warp factors Z_1, Z_2, Z_3 correspond to D0, D4 and F1 charges respectively.

When the four-dimensional base of the solution is a multi-center Gibbons–Hawking (Taub–NUT) space,

$$ds_4^2 = V^{-1}(d\psi + A)^2 + V d\vec{y} \cdot d\vec{y}, \tag{4}$$

the full solution can be determined in terms of eight harmonic functions, V, K^I, L_I, M ($I = 1, 2, 3$) on the \mathbb{R}^3 spanned by (y_1, y_2, y_3) . As shown in [20], the RR potentials are given by

$$C^{(1)} = (Z_1^{-1} - 1)dt + Z_1^{-1}k - \zeta^{(1)}, \quad (5)$$

$$C^{(3)} = -Z_3^{-1}(dt + k) \wedge \zeta^{(1)} \wedge dx_5 - dt \wedge A^{(3)} \wedge dx_5 + (\nu_a + V^{-1}K^3\xi_a^{(1)})\Omega_-^{(a)} \wedge dx_5, \quad (6)$$

where

$$\vec{\nabla} \times \vec{\nu} = -\vec{\nabla} L_2 \quad \text{and} \quad \Omega_-^{(a)} = (d\psi + A) \wedge dy^a - \frac{1}{2}\epsilon_{abc}dy^b \wedge dy^c. \quad (7)$$

$$\zeta^{(I)} = V^{-1}K^I(d\psi + A) + \xi^I, \quad \vec{\nabla} \times \vec{\xi}^I \equiv -\vec{\nabla} K^I. \quad (8)$$

The functions, Z_I , and the angular momentum one-form, k , are

$$Z_I = \frac{1}{2}V^{-1}C_{IJK}K^JK^K + L_I, \quad k = \mu(d\psi + A) + \omega, \quad (9)$$

where

$$\mu = \frac{1}{6}V^{-2}C_{IJK}K^IK^JK^K + \frac{1}{2}V^{-1}K^IL_I + M. \quad (10)$$

and the one form, $\omega = \vec{\omega} \cdot d\vec{x}$, is given by the solution of the equation

$$\vec{\nabla} \times \vec{\omega} = V\vec{\nabla}M - M\vec{\nabla}V + \frac{1}{2}(K^I\vec{\nabla}L_I - L_I\vec{\nabla}K^I). \quad (11)$$

We take the harmonic functions to have the form:

$$V = \epsilon_0 + \sum_{j=1}^N \frac{q_j}{r_j}, \quad K^I = \kappa_0^I + \sum_{j=1}^N \frac{k_j^I}{r_j}, \quad (12)$$

$$L_I = l_0^I + \sum_{j=1}^N \frac{l_j^I}{r_j}, \quad M = m_0 + \sum_{j=1}^N \frac{m_j}{r_j}, \quad (13)$$

where $r_j = |\vec{y} - \vec{y}_j|$, for N Gibbons-Hawking (GH) points located at \vec{y}_j . To ensure that the solution is regular (up to \mathbb{Z}_{q_j} orbifold singularities) at $r_j \rightarrow 0$ we must have $q_j \in \mathbb{Z}$ and

$$l_j^I = -\frac{1}{2}q_j^{-1}C_{IJK}k_j^Jk_j^K, \quad m_j = \frac{1}{2}q_j^{-2}k_j^1k_j^2k_j^3, \quad j = 1, \dots, N. \quad (14)$$

As shown in [13], the spectral flow transformation:

$$\tilde{L}_I = L_I - 2\gamma_I M, \quad \tilde{M} = M, \quad \vec{\tilde{\omega}} = \vec{\omega}, \quad (15)$$

$$\tilde{K}^I = K^I - C^{IJK}\gamma_J L_K + C^{IJK}\gamma_J \gamma_K M, \quad (16)$$

$$\tilde{V} = V + \gamma_I K^I - \frac{1}{2}C^{IJK}V\gamma_I \gamma_J L_K + \frac{1}{3}C^{IJK}\gamma_I \gamma_J \gamma_K M, \quad (17)$$

transforms solution to solutions, and can change a GH centers into other GH centers, or two-charge supertubes³. This can be arranged to happen at the N^{th} GH point by choosing:

$$\gamma_{1,3} = 0, \quad \gamma_2 = \gamma = -\frac{q_N}{k_N^2}, \quad (18)$$

³From a four-dimensional perspective this corresponds to transforming a primitive D6 brane into a primitive D4 brane.

which induces the following changes:

$$\begin{aligned}\tilde{Z}_1 &= \frac{V}{\tilde{V}} Z_1, & \tilde{Z}_3 &= \frac{V}{\tilde{V}} Z_3, & \tilde{Z}_2 &= \frac{\tilde{V}}{V} Z_2 - 2\gamma\mu + \gamma^2 \frac{Z_1 Z_3}{\tilde{V}}, \\ \tilde{\mu} &= \frac{V}{\tilde{V}} \left(\mu - \gamma \frac{Z_1 Z_3}{\tilde{V}} \right), & \tilde{V} &= V + \gamma K^2.\end{aligned}\tag{19}$$

As explained in [13], the dipole charge and “bare” electric charges of the resulting supertube are given by the coefficients of the divergent terms in \tilde{K}^2 , \tilde{L}_1 and \tilde{L}_3 . We define the “effective” charges of the supertube by the divergence of the electric potentials, Z_I , near the supertube:

$$Q_1^{eff} \equiv 4 \lim_{r_N \rightarrow 0} r_N \tilde{Z}_1 = 4q_N (\tilde{V}^{-1} Z_1)|_{r_N=0} = 4\tilde{\ell}_N^1 + 4\tilde{k}_N^2 (\tilde{V}^{-1} \tilde{K}^3)|_{r_N=0}, \tag{20}$$

and similarly for Q_3^{eff} . As we will see later, these are the charges that determine the entropy of supertubes, and since $(\tilde{V}^{-1} \tilde{K}^3)$ depends critically on the position of the supertube, the effective charges can be much larger than the asymptotic charges of the system. This is the crucial ingredient of the entropy enhancement mechanism.

3 The probe calculation

Consider a probe supertube with D0 and F1 charges and D2 dipole charge in the three-charge solution with a Gibbons-Hawking base described above. We choose the supertube world-volume coordinates ξ to be $(t, \theta = \psi, z = x^5)$, where ψ is the $U(1)$ fiber of the GH base.

The DBI–WZ action of the supertube is:

$$\begin{aligned}S = T_{D2} \int d^3\xi \left\{ \left[\left(\frac{1}{Z_1} - 1 \right) \mathcal{F}_{z\theta} + \frac{K^3}{Z_1 V} + (\mathcal{F}_{tz} - 1) \left(\frac{\mu}{Z_1} - \frac{K^1}{V} \right) \right] \right. \\ \left. - \left[\frac{1}{V^2 Z_1^2} [(K^3 - V(\mu(1 - \mathcal{F}_{tz}) - \mathcal{F}_{z\theta}))^2 + V Z_1 Z_2 (1 - \mathcal{F}_{tz})(2 - Z_3(1 - \mathcal{F}_{tz}))] \right]^{1/2} \right\}, \tag{21}\end{aligned}$$

where $2\pi\alpha' F \equiv \mathcal{F} = \mathcal{F}_{tz} dt \wedge dz + \mathcal{F}_{z\theta} dz \wedge d\theta$ is the world-volume gauge field of the D2 brane. Our goal is to semi-classically quantize BPS fluctuations around certain supertube configurations, and compute their entropy. Supersymmetry requires that these fluctuations be independent of t and z , and that $\mathcal{F}_{tz} = 1$.

All the fluctuations of the supertube lead to similar values for the entropy, but for the purpose of illustrating entropy enhancement it is best to focus on the fluctuations in the four torus directions:

$$x_i \rightarrow x_i + \eta_i(t, \theta) \quad i = 1 \dots 4. \tag{22}$$

Since the BPS modes are independent of z , it is convenient to work with a Lagrangian density that has already been integrated over the z direction, which gives the conjugate momenta for the excitations η_i :

$$\Pi_i = \left(\frac{\partial}{\partial \dot{\eta}_i} \int_0^{2\pi L_z} \mathcal{L}_{WZ} + \mathcal{L}_{DBI} \right)_{\dot{\eta}_i=0, \mathcal{F}_{tz}=1} = 2\pi L_z T_{D2} \eta'_i, \tag{23}$$

where $\dot{\eta}_i \equiv \frac{\partial \eta_i}{\partial t}$ and $\eta'_i \equiv \frac{\partial \eta_i}{\partial \theta}$. To semi-classically quantize the BPS oscillations we impose the canonical commutation relations:

$$[\eta_j(t, \theta), \Pi_k(t, \theta')] = i\delta_{jk}\delta(\theta - \theta'). \quad (24)$$

A supertube with dipole charge n_2 can be thought of as wrapped n_2 times around the θ circle. To find the correct mode expansion it is not enough to focus on the BPS modes alone, even if one only wants to count the entropy coming from these modes. Both the BPS and non-BPS modes contribute to the delta-function in (24) and the inclusion of both contributions is essential to the proper normalization of the modes⁴. The result is simply an extra factor of $\sqrt{2}$ in the coefficient of the BPS mode expansion compared to the naive expansion that neglects non-BPS modes:

$$\eta_i = \eta_i^{\text{BPS}} + \eta_i^{\text{nonBPS}} = \frac{1}{\sqrt{8\pi^2 T_{D2} L_z}} \sum_{k>0} \left[e^{ik\theta/n_2} \frac{(a_k^i)^\dagger}{\sqrt{|k|}} + \text{h.c.} \right] + \eta_i^{\text{nonBPS}}. \quad (25)$$

The creation and annihilation operators, $(a_k^i)^\dagger$ and a_k^i , for the modes in the k^{th} harmonic satisfy canonical commutation relations:

$$[a_k^i, (a_{k'}^j)^\dagger] = \delta^{ij}\delta_{k,k'}. \quad (26)$$

The D0 and F1 quantized charges of the supertube are:

$$Q_1 = \frac{T_{D2}}{T_{D0}} \int_0^{2\pi L_z} dz \int_0^{2\pi n_2} d\theta \mathcal{F}_{z\theta} = 4\pi^2 \frac{T_{D2}}{T_{D0}} L_z n_2 \mathcal{F}_{z\theta} \quad (27)$$

$$Q_3 = \frac{T_{D2}}{T_{F1}} \int_0^{2\pi n_2} d\theta \left[-\frac{K^1}{V} + \frac{1}{\mathcal{F}_{z\theta} + V^{-1}K^3} \left(\frac{Z_2}{V} + (\eta')^2 \right) \right] \quad (28)$$

Substituting (25) into (28) and rearranging using (27) leads to:

$$\begin{aligned} \sum_{i=1}^4 \sum_{k>0} k (a_k^i)^\dagger a_k^i &= L_z T_{D2} \int_0^{2\pi n_2} d\theta \int_0^{2\pi n_2} d\theta' \sum_{i=1}^4 \eta'_i \eta_i \\ &= \left[Q_1 + 2\pi T_{F1} L_z n_2 \frac{K^3}{V} \right] \left[Q_3 + \frac{2\pi T_{D2}}{T_{F1}} n_2 \frac{K^1}{V} \right] - 4\pi^2 T_{D2} L_z n_2^2 \frac{Z_2}{V}, \end{aligned} \quad (29)$$

where the integrals over θ and θ' are precisely those appearing in each of (27) and (28). This is the result we have been seeking. The left hand side of (29) can be thought of as the total energy L_0 of a set of four harmonic oscillators in 1 + 1 dimensions. For large L_0 , the entropy coming from the different ways of distributing this energy to various modes of these oscillators is given by the Cardy formula:

$$S = 2\pi \sqrt{\frac{c L_0}{6}}. \quad (30)$$

⁴This subtlety is correctly taken into account in [10], but not in [11].

Since we count BPS excitations, there will be also 4 fermionic degrees of freedom, and the central charge associated to the torus oscillations will be $c = 4 + 2 = 6$, giving the entropy:

$$S = 2\pi \sqrt{\left[Q_1 + n_2 \frac{K^3}{V}\right] \left[Q_3 + n_2 \frac{K^1}{V}\right] - n_2^2 \frac{Z_2}{V}} = 2\pi \sqrt{Q_1^{eff} Q_3^{eff} - n_2^2 \frac{Z_2}{V}}, \quad (31)$$

where to render the equations simple we have chosen a system of units in which $2\pi T_{F1} L_z = L_z/\alpha' = 1$ and $2\pi T_{D2}/T_{F1} = (g_s \sqrt{\alpha'})^{-1} = 1$. We will use this convention throughout this letter. Equation (29) has two important consequences. First, for a supertube with a given set of BPS modes, this equation is nothing but a “radius formula” that determines its size by fixing, in the spatial base, the location of the $U(1)$ fiber that it wraps. When the supertube is maximally spinning, and has no BPS modes, this equation simply becomes the radius formula of the maximally spinning supertube [20]. The second result is that this formula also determines the capacity of the supertube to store entropy: In flat space, this capacity is determined by the asymptotic charges, Q_1 and Q_3 , whereas, in a more general background, the capacity to store entropy is determined by Q_1^{eff} and Q_3^{eff} . In certain backgrounds, the latter can be made much larger than the former and so a supertube of given asymptotic charges can have a lot more modes and thus store a lot more entropy by the simple expedient of migrating to a location where the effective charges are very large. We will discuss this further below.

Clearly, for bubbling backgrounds, and even for black ring backgrounds, the right hand side of (29) can diverge, and one naively gets an infinite value for the entropy. Nevertheless, as we mentioned in the introduction, this calculation is done in the approximation that the supertube does not back-react on the background, and taking this back-reaction into account will modify this naive conclusion.

For a supertube that is not along the GH fiber, equation (31) is still correct, except that the Q_I^{eff} are no longer given by (20) but by:

$$Q_1^{eff} \equiv Q_1 + n_2 \tilde{\zeta}^{(1)}, \quad Q_3^{eff} \equiv Q_3 + n_2 \tilde{\zeta}^{(2)}. \quad (32)$$

where $\tilde{\zeta}^{(I)}$ are the pull-backs onto the supertube of the spacetime one-forms $\zeta^{(I)}$ defined in (8).

We have also explicitly calculated the supertube entropy in a general three-charge black-ring background, where the supertube oscillates both in the T^4 , and in two of the transverse \mathbb{R}^4 directions. The result is identical to (31), except that now there are six possible bosonic modes (and thus after we include the corresponding fermions the central charge of the system is $c = 9$). The explicit answer for the entropy⁵ is:

$$S = 2\pi \sqrt{\frac{cL_0}{6}} = 2\pi \sqrt{\frac{3}{2}} \sqrt{[(Q_1 - 2n_2 q_3(1+y))][Q_3 - 2n_2 q_1(1+y)] - n_2^2 Z_2 R^2 \frac{(y^2 - 1)}{(x - y)^2}}, \quad (33)$$

Based on this result, we expect that upon including the four bosonic shape modes in the transverse space, as well as the fermionic counterparts of all the eight bosonic modes, the central charge c should jump from 6 to 12, and equation (31) to be modified accordingly. We have also explicitly computed the entropy coming from arbitrary shape modes, and the formulas do

⁵Using the conventions of [2].

display entropy enhancement (they diverge near $y = -\infty$ for the black ring). However, the complete expressions are rather unilluminating, and we leave their study for later investigation [20]. Our calculation agrees with the entropy of supertubes in flat space-time, computed using similar methods in [10, 11], and using different methods in [12].

It is also possible to compute the angular momentum of a supertube that has a very large number of BPS modes turned on. From the T_{0i} components of the energy momentum tensor we find

$$J^{ij} = \frac{1}{2\pi} \int_0^{2\pi n_2} d\theta (\eta_i \Pi_j - \eta_j \Pi_i) \quad (34)$$

and the angular momentum of the tube along the GH fiber is

$$J = \frac{Q_1 Q_3}{n_2} - \frac{Q_1^{eff} Q_3^{eff}}{n_2} + n_2 \frac{Z_2}{V}. \quad (35)$$

From this identity we may simply re-write (31) as

$$S = 2\pi \sqrt{Q_1^{eff} Q_3^{eff} - n_2^2 \frac{Z_2}{V}} = 2\pi \sqrt{Q_1 Q_3 - n_2 J}. \quad (36)$$

Hence, in a certain sense, (31) is the same as the entropy formula for a supertube in empty space and it naively appears that entropy enhancement has gone away. It has not. The important point is that (35) implies that it is possible for J to become extremely large and negative as the number of BPS modes on the tube increases⁶. In flat space, $|J|$ is limited by $|Q_1 Q_3|$ but in a general background our Born-Infeld analysis (equations (29) and (35)) imply that the upper bound is the same but there is no lower bound.

From the supergravity perspective, the limits on J usually emerge from requiring that there are no CTC's near the supertube. This is a local condition set by the local behavior of the metric, and particularly by the Z_I , near the supertube. Although we do not have the explicit solution, our analysis suggests that the lower limit of the angular momentum of the supertube is controlled by Q_1^{eff} and Q_3^{eff} as opposed to Q_1 and Q_3 . Thus entropy enhancement can occur if the supertube moves to a region where Q_1^{eff} and Q_3^{eff} are extremely large and then a vast number of modes can be supported on a supertube (of fixed Q_1 and Q_3) by making J large and negative. We therefore expect the corresponding supergravity solution to be CTC-free provided that $|n_2 J| < Q_1^{eff} Q_3^{eff}$.

One should thus think of a supertube of given n_2 , Q_1 and Q_3 as being able to store a certain number of modes before it over-spins. The “storage capacity” of the supertube is determined by the local conditions around the supertube and, specifically, by n_2 , Q_1^{eff} and Q_3^{eff} . Magnetic dipole interactions, like those evident in bubbling backgrounds, can thus greatly modify the capacity of a given supertube to store entropy.

⁶This is not unexpected: As in flat space, every BPS mode on the supertube takes away one quantum of angular momentum of the tube.

4 Entropy Enhancement - the Proposal

As we have seen, the entropy of a supertube, and hence the entropy of a fluctuating geometry, depends upon the *local* effective charges and not upon the asymptotic charges measured at infinity. In the derivation of (29) we started with a maximally spinning, round supertube with zero entropy and perturbed around it. For the maximally spinning tube, the equilibrium position is determined by the vanishing of the right-hand side. Upon adding wiggles to the tube, the right hand side no longer vanishes and the imperfect cancelation is responsible for the entropy.

It is interesting to ask how much entropy can equation (29) accommodate. The answer is not so simple. At first glance one might say that the both terms in the right hand side of (29) can be divergent, and hence the entropy of the fluctuating tube is infinite. Nevertheless, one can see that the leading order divergent terms in $Q_1^{eff} Q_3^{eff}$ and in $n_2^2 Z_2/V$ come entirely from bulk supergravity fields, and exactly cancel, both for the supertube in GH background and for the supertube near a black ring (33).

It is likely that this partial cancelation is an artefact of the extremely symmetric form of the solution, and that in a more general solution such cancellation may not take place. In particular, both Q_1^{eff} and Q_3^{eff} are integrals of “effective charge” densities on the supertube world-volume, and the right hand side of equation (29) should be written as

$$Q_1^{eff} Q_3^{eff} - n_2^2 \frac{Z_2}{V} = \int \rho_1^{eff} d\theta \int \rho_3^{eff} d\theta - \int \rho_1^{eff} \rho_3^{eff} d\theta \quad (37)$$

If this generalized formula is correct, certain density and shape modes will disturb the balance between the product of integrals and the integral of the product, and the leading behavior of the entropy will still be of the order

$$S \sim \sqrt{Q_1^{eff} Q_2^{eff}}. \quad (38)$$

Regardless of this, the next-to-leading divergent terms in (31) are a combination of supertube world-volume terms and bulk supergravity fields. In a scaling solution, or when the tube is close to the black ring, these terms can diverge, giving naively an infinite entropy. As we discussed above, we expect the back-reaction of the supertubes to render this entropy finite.

The idea of *entropy enhancement* is that one can find backgrounds in which the effective charges of a two-charge supertube can be made far larger than the asymptotic charges of the solution, and that, in the right circumstances, the oscillations of this humble supertube could give rise to an entropy that grows with the asymptotic charges much faster than $\sqrt{Q^2}$ (as typical for supertubes), and might even grow as fast as $\sqrt{Q^3}$, as typical for black holes in five dimensions.

To achieve such a vast enhancement requires a very strong magnetic dipole-dipole interaction and this means that multiple magnetic fluxes must be present in the solution. It is *not* sufficient to have a large red-shift: BMPV black holes have infinitely long throats and arbitrarily large red-shifts but have no magnetic dipole moments to enhance the effective charges and thus increase the entropy that may be stored on a given supertube.

Hence, the obvious places to obtain entropy enhancement are solutions with large dipole magnetic fields, such as black ring or bubbling microstate solutions. Since we are focussing on trying to obtain the entropy of black holes from horizonless configurations, we will focus on the latter. These bubbling solutions are constructed using an ambi-polar base GH metric, and near

the “critical surfaces,” where V vanishes, the term $\frac{K^I}{V}$ in the effective charge diverges. It is therefore natural to expect entropy enhancement for supertubes that localize near the critical ($V = 0$) surfaces.

We also believe that placing supertubes in deep scaling solutions [4, 5, 19] will prove to be an equally crucial ingredient. Indeed, as we will see in the next section, in a deep microstate geometry the K^I at the location of the tube can also become large, and hence there will be a double enhancement of the effective charge, both because of the vanishing V in the denominator and because of the very large K^I in the numerator. There is another obvious reason for this: It is only the scaling microstate geometries that have the same quantum numbers as black holes with macroscopic horizons.

This must mean that the simple entropy enhancement one gets from the presence of critical surfaces is not sufficient for matching the black hole entropy. The fundamental reason for this may well be the following: Even if the round supertube can be brought very close to the $V = 0$ surface, once the supertube starts oscillating it will necessarily sample the region around this surface, and the charge enhancement will correspond to the average Q_I^{eff} in that region. For this to be very large the entire region where the supertube oscillates must have a very significant charge enhancement. The only such region in a horizonless solution is the bottom of a deep or scaling throat, where the average of the K^I is indeed very large.

All the issues we have raised here have to do with the details of the entropy enhancement mechanism, and involve some very long and complex calculations that we intend to pursue in future work. We believe their clarification is very important, as it will shed light on how the entropy of black holes can be realized at the level of horizonless configurations.

Our goals in this letter are rather more modest. We have shown via a Born-Infeld probe calculation that the entropy of supertubes is given by their effective charges, and not by their brane charges, and that these effective charges can be very large. However, because the supertube has been treated as a probe in our calculations, it is logically possible that, once we take into account its back-reaction, the bubble equations may forbid the supertube to get suitably close to the $V = 0$ surfaces, and to have a suitable entropy enhancement.

In principle this is rather unlikely, as we know that in all the examples studied to date, the solutions of the Born-Infeld action of supertubes always correspond to configurations that are smooth and regular in supergravity [20]. However, settling the issue completely is not possible before constructing the full supergravity solutions corresponding to wiggly supertubes. Hence, in the remainder of this letter we will show that at least for the maximally-spinning supertubes, their effective charges in deep scaling solutions can lead to a black-hole-like enhanced entropy.

5 Supertubes in scaling microstate geometries

To find bubbling solutions that contain supertubes with enhanced charges one could look for solutions of the bubble or integrability equations [16, 17, 6]

$$\sum_{\substack{j=1 \\ j \neq i}}^N \frac{\Gamma_{ij}}{r_{ij}} = 2 (\epsilon_0 m_i - m_0 q_i) + \sum_{I=1}^3 (\ell_i^I \kappa_0^I - \ell_0^I k_i^I) , \quad r_{ij} \equiv |\vec{y}^{(i)} - \vec{y}^{(j)}| \quad (39)$$

that describe scaling solutions where some of the centers are GH points, and the other centers are supertubes. However, it is more convenient to construct such solutions by spectrally flowing multi-center GH solutions, which have been studied much more. The parameters of the equations are then:

$$\Pi_{ij}^{(I)} \equiv \left(\frac{k_j^I}{q_j} - \frac{k_i^I}{q_i} \right), \quad \Gamma_{ij} = q_i q_j \Pi_{ij}^{(1)} \Pi_{ij}^{(2)} \Pi_{ij}^{(3)}. \quad (40)$$

One obtains a scaling solution when a subset, \mathcal{S} , of the GH points approach one another arbitrarily closely, that is, $r_{ij} \rightarrow 0$ for $i, j \in \mathcal{S}$. In terms of the physical geometry, these points are remaining at a fixed distance from each other, but are descending a long AdS throat that, in the intermediate region, looks almost identical to the throat of a black hole or black ring (depending upon the total GH charge in \mathcal{S}). In particular, in the intermediate regime, one has $Z_I \sim \frac{\hat{Q}_I}{4r}$, where we have taken \mathcal{S} to be centered at $r = 0$ and the \hat{Q}_I are the electric charges associated with \mathcal{S} . Similarly, if \mathcal{S} has a non-zero total GH charge of \hat{q}_0 , then one has $V \sim \frac{\hat{q}_0}{r}$. More precisely:

$$Z_I V = \ell_0^I V + \varepsilon_0 (L_I - \ell_0^I) - \frac{1}{4} C_{IJK} \sum_{i,j=1}^N \Pi_{ij}^{(J)} \Pi_{ij}^{(K)} \frac{q_i q_j}{r_i r_j}. \quad (41)$$

Suppose that we perform a spectral flow so that some point, $p \in \mathcal{S}$, becomes a supertube. Let \tilde{V}_p be the value of \tilde{V} at p . Then, from (19) and (20), the effective charges of this supertube are dominated by terms from interactions with the magnetic fluxes in the throat:

$$Q_I^{eff} \sim -2 q_p \tilde{V}_p^{-1} C_{IJK} \sum_{j \in \mathcal{S}, j \neq p} \Pi_{jp}^{(J)} \Pi_{jp}^{(K)} \frac{q_j}{r_{jp}}. \quad (42)$$

However, observe that $\tilde{q}_j = (k_p^2)^{-1} q_p q_j \Pi_{jp}^{(2)}$ and so

$$q_p^{-1} \tilde{V}_p \sim (k_p^2)^{-1} \sum_{j \in \mathcal{S}, j \neq p}^N \frac{q_j \Pi_{jp}^{(2)}}{r_{jp}}. \quad (43)$$

Therefore the numerator and denominator of (42) have the same naive scaling behavior as $r_{jp} \rightarrow 0$ and so, in general, Q_I^{eff} will attain a finite limit that only depends upon the q_j, k_j^I for $j \in \mathcal{S}$. Indeed, the finite limit of Q_I^{eff} scales as the square of the k 's for large k_j^I parameters. This is no different from the typical values of asymptotic electric charges in bubbled geometries.

However, since we are in a bubbled microstate geometry, V and \tilde{V} change sign throughout the bubbled region. In particular, there are surfaces at the bottom of the throat where \tilde{V} vanishes and there are regions around them where \tilde{V} remains finite and bounded as $r_{ij} \rightarrow 0$. Suppose that we can arrange for the supertube point p to be in such a region of a scaling throat and at the same time we can arrange that Z_I still diverges as $\frac{1}{r}$. Then, in principle, the effective charges, of the supertube Q_I^{eff} , could become arbitrarily large.

As mentioned above, we expect the entropy of the system to come from wiggly supertubes in throats that are neither very deep (to allow the tubes to wiggle), nor very shallow (to give enhancement). We do not, as yet, know how to take the back-reaction of the wiggly supertubes into account, and hence we do not have any supergravity argument about the length of these

throats. However, we can use the *AdS*-CFT correspondence and the fact that we know what the typical CFT microstates are, to argue [4] that the typical bulk microstates are scaling solutions that have GH size r_T given by

$$r_T \sim \bar{Q}^{-1/2} \sim \frac{1}{\bar{k}}, \quad (44)$$

where \bar{Q} is the charge and \bar{k} is the typical flux parameter.

If one takes this *AdS*-CFT result as given, and moreover assumes that the wiggling supertube remains in a region of finite \tilde{V} in the vicinity of the $\tilde{V} = 0$ surface, one then has:

$$Q_I^{eff} \sim (\bar{k})^3 \sim \bar{Q}^{3/2} \quad (45)$$

because $\Pi_{jp}^{(K)} \sim \bar{k}$, and hence the entropy of the fluctuating supertube (38) would depend upon the asymptotic charges as:

$$S \sim \sqrt{Q_1^{eff} Q_2^{eff}} \sim \bar{Q}^{3/2}. \quad (46)$$

which is precisely the correct behavior for the entropy of a classical black hole!

These simple arguments indicate that fluctuating supertubes at the bottom of deep scaling microstate geometries can give rise to a black-hole-like macroscopic entropy, provided that they oscillate in a region of bounded \tilde{V} .

Obviously there is a great deal to be checked in this argument, particularly about the effect of the back-reaction of the supertube on its localization near the $\tilde{V} = 0$ surface. We conclude this section by demonstrating that at least maximally spinning tubes, for which we can construct the supergravity solution, have no problem localizing in a region of finite \tilde{V} . As the solution scales, the effective charges diverge, as is needed for entropy enhancement.

5.1 An example

One can construct a very simple deep scaling solution using three Taub-NUT (GH) centers with charges q_1, q_2 and q_3 , and fluxes arranged so that the $|\Gamma_{ij}|$, $i, j = 1, 2, 3$, satisfy the triangle inequalities. The GH points then arrange themselves asymptotically as a scaled version of this triangle:

$$r_{ij} \rightarrow \lambda |\Gamma_{ij}|, \quad \lambda \rightarrow 0. \quad (47)$$

One can then take a spectral-flow of this solution so that the second GH point becomes a two-charge supertube. For simplicity, we will choose $q_1 \Pi_{12}^{(2)} = q_3 \Pi_{23}^{(2)}$ so that after the flow the GH charges of the remaining two GH points will be equal and opposite:

$$\tilde{q}_1 = -\tilde{q}_3. \quad (48)$$

For \tilde{V}_p to remain finite in the scaling limit, the supertube must approach the plane equidistant from the remaining GH points.

We have performed a detailed analysis of such solutions and used the absence of CTC's close to the GH points, in the intermediate throat and in the asymptotic region to constrain the possible fluxes. We have found a number of such solutions that have the desired scaling properties for Q_I^{eff} and we have performed extensive numerical analysis to check that there are

no regions with CTC's. In particular, we checked numerically that the inverse metric component, g^{tt} , is globally negative and thus the metric is stably causal. We will simply present one example here.

Consider the asymptotically Taub-NUT solution with:

$$q_1 = 16, \quad q_2 = 96, \quad q_3 = -40, \quad \epsilon_0 = 1, \quad Q_0 \equiv q_1 + q_2 + q_3 = 72 \quad (49)$$

and

$$k_1^I = (8, -88, 8), \quad k_2^I = (0, 96, 0), \quad k_3^I = (20, 64, 20), \quad (50)$$

where Q_0 is the KK monopole charge of the solution. With these parameters one has the following fluxes:

$$\Pi_{12}^{(I)} = (-\frac{1}{2}, \frac{13}{2}, -\frac{1}{2}), \quad \Pi_{23}^{(I)} = (-\frac{1}{2}, -\frac{13}{5}, -\frac{1}{2}), \quad \Pi_{13}^{(I)} = (-1, \frac{39}{10}, -1), \quad (51)$$

and

$$\Gamma_{12} = \Gamma_{23} = \Gamma_{31} = 2496. \quad (52)$$

In this scaling solution the GH points form an equilateral triangle and thus, after the spectral flow, the supertube will tend to be equidistant from the two GH points of equal and opposite charges (48), and therefore will approach the surface where $\tilde{V} = 0$.

The solution to the bubble equations yields

$$r_{12} = \frac{11232 r_{13}}{11232 + 359 r_{13}}, \quad r_{23} = \frac{11232 r_{13}}{11232 + 731 r_{13}}, \quad (53)$$

which satisfies the triangle inequalities for $r_{13} \leq \frac{11232}{\sqrt{262429}} \approx 21.9$. After spectral flow the value of \tilde{V} at the location of the supertube (point 2) is

$$\tilde{V}_2 = 1 + \frac{104}{r_{12}} - \frac{104}{r_{23}} = -\frac{22}{9}, \quad (54)$$

independent of r_{13} . In particular, it remains finite and bounded as the three points scale and the distances between them go to zero. The effective charges of the supertube are given by

$$Q_1^{eff} = Q_3^{eff} = 384 \tilde{V}_2^{-1} \left(1 + \frac{52}{r_{12}} + \frac{52}{r_{23}} \right), \quad (55)$$

and scale as λ^{-1} as $\lambda \rightarrow 0$ in (47). We thus have effective charges that naively scale to arbitrarily large values. As described earlier, we expect this scaling to stop as the supertubes become more and more wiggly, and we expect the entropy to come from configurations of intermediate throat depth.

Finally, this configuration has asymptotic electric, and Kaluza-Klein charges:

$$Q_1 = 416, \quad Q_2 = \frac{608}{9}, \quad Q_3 = 416, \quad J_R = Q_{KK}^E = \frac{5824}{9}, \quad Q_0 = Q_{KK}^M = 72. \quad (56)$$

and is thus a microstate of a Taub-NUT black hole with a finite extremality parameter and a macroscopic horizon:

$$\frac{Q_0 Q_1 Q_2 Q_3 - \frac{1}{4} Q_0^2 J_R^2}{Q_0 Q_1 Q_2 Q_3} = \frac{27}{76} \approx 36\%. \quad (57)$$

6 Conclusions

The most important result presented in this letter is that the entropy of a supertube in a given background is not determined by its charges, but rather by its “effective charges,” which receive a contribution from the interaction of the magnetic dipole moment of the tube with the magnetic fluxes in the background. As a result, one can get very dramatic *entropy enhancement* if a supertube is placed in a suitable background. We have argued that this enhancement can give rise to a macroscopic (black-hole-like) entropy, coming entirely from smooth horizonless configurations.

Three ingredients are needed for this dramatic entropy enhancement:

- (i) Deep or scaling solutions
- (ii) Ambi-polar base metrics
- (iii) BPS fluctuations that localize near the critical ($V = 0$) surfaces of the ambi-polar metrics

These are also precisely the ingredients that have emerged from recent developments in the study of finite-sized black-hole microstates in the regime of parameters where the gravitational back-reaction of some of the branes is negligible. Indeed, deep scaling ambi-polar configurations are needed both to get a macroscopic entropy in the “quiver quantum mechanics regime” [19], and to get smooth microstates of black holes with macroscopic horizons [4]. Furthermore, the D0 branes that can give a black-hole-like entropy in a D6- $\overline{\text{D6}}$ background [18] must localize near the critical surface of the ambi-polar base, much like the supertubes in our analysis. It would be fascinating to find a link between the microscopic configurations constructed in these papers, and those we consider here.

In this letter we have referred to the entropy enhancement mechanism as a “proposal” because a number of the details need to be carefully checked by careful computation. Most importantly, we have performed a classical calculation using a brane probe near a critical surface. It is important to study the fluctuating, or wiggling, supertubes in the full supergravity theory and determine how the back-reaction of the fluctuations modifies the picture presented here. One important issue is whether fluctuating supertubes can still remain in the region close to the critical surface with V finite and bounded. Another is to understand the interplay between how much a supertube wiggles and how long its throat can get or how much the supergravity solution it sources can scale.

While some of the details need to be explored very carefully, we believe that the mechanism and the approach given in this paper may well provide the key to understanding how fluctuating microstate geometries can provide a semi-classical description of black-hole entropy in the regime of parameters where the black hole exists.

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